Sydney Technical High School

Name: ....

Maths Class: ....

Year 12

Mathematics Extension 1

HSC Course

Assessment 1

December, 2020

Time allowed: 90 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A reference sheet is provided.

Section 1 Multiple Choice
Questions 1-7
7 Marks

Section II Questions 8-12
60 Marks

Section I

7 marks

Attempt Questions 1-7

Allow about 10 minutes for this section

Use the multiple choice answer sheet for Questions 1-7.

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1. The polynomial $P(x) = x^3 - 4x^2 - 6x + k$ has a factor of $x - 2$.
   What is the value of k?

   (A) 2

   (B) 12

   (C) 20

   (D) 36

2. Which of the following is a simplification of $4log_e\sqrt{e^x}$?

   (A) $\frac{1}{2}x$

   (B) $2x$

   (C) x^2

   (D) $4\sqrt{x}$

3. How many arrangements of the letters of the word 'OLYMPIC' are possible if the C and L are to be together?

   (A) 120

   (B) 240

   (C) 720

   (D) 1440

4. If $\frac{dP}{dt} = 0.2(P - 10)$ and $P = 30$ when $t = 0$, which of the following is an expression for P?

   (A) $P = 10 + 20e^{0.2t}$

   (B) $P = 20 + 10e^{0.2t}$

   (C) $P = 20 + 30e^{0.2t}$

   (D) $P = 30 + 20e^{0.2t}$

5. Given that $f(x) = e^x - 1$, and $y = f^{-1}(x)$, find an expression for $\frac{dy}{dx}$

(A) $\frac{1}{e^x - 1}$

(B) $\frac{1}{x+1}$

(C) lnx

(D) $\ln(x+1)$

6. A body of still water has suffered an oil spill and a circular oil slick is floating on the surface of the water.
The area of the oil slick is increasing by $0.1\ m^2$/minute. At what rate is the radius increasing when the radius is 0.3?

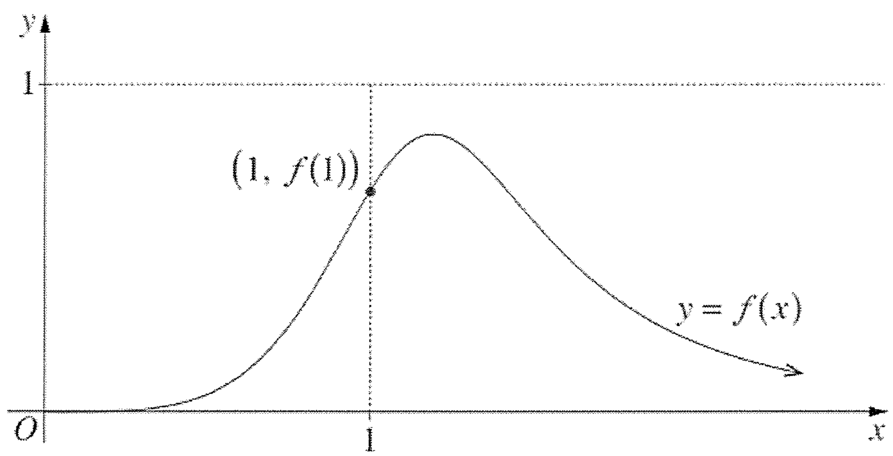
(A) 0.161 m/minute

(B) 0.03 m/minute

(C) 0.0515 m/minute

(D) 0.0531 m/minute

7. The diagram shows the graph of $y = f(x)$.
You can assume that the marked point is **not** a point of inflexion.



Which of the following is a correct statement?

(A) $f''(1) < f(1) < 1 < f'(1)$

(B) $f''(1) < f'(1) < f(1) < 1$

(C) $f(1) < 1 < f'(1) < f''(1)$

(D) $f'(1) < f(1) < 1 < f''(1)$

Section II

Total marks – 60

Attempt Question 8-12

Allow about 1 hour and 20 minutes for this section

Begin each question on a NEW page

In Questions 8-12, your responses should include relevant mathematical reasoning and/or calculations.

Question 8 (12 marks)

a) Differentiate the following with respect to x

(i) $y = e^x log_e(2x)$ — 1

(ii) $y = 6^{5x}$ — 1

(iii) $y = ln\left(\frac{4x+1}{3x-2}\right)$ — 2

b) Ron, Harry and seven friends arrange themselves around a circular table.

(i) In how many ways can they be arranged? — 1

(ii) If they are seated randomly, what is the probability that Ron and Harry are not sitting next to each other? — 2

c) Solve the equation $3^{x-1} = 5$ correct to 2 decimal places — 2

d) Solve $(n+2)! = 72n!$ — 3

End of Question 8

Question 9 (12 marks)

a) Find $\frac{d}{dx}(log_7 x^2)$ 1

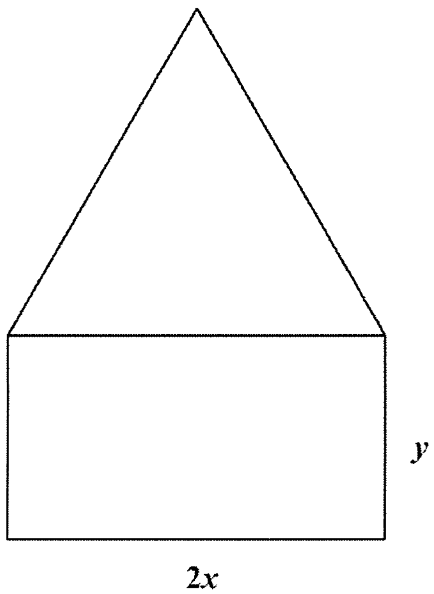
b) A set of 20 students is made up of 10 students from Year 11 and 10 students from Year 12. Five students are to be selected from the set of 20. The order is unimportant.

(i) What is the total number of possible selections? 1

(ii) What is the total number of selections, if there are at least two Year 11 students and at least 2 Year 12 students in the group of 5? 1

c) Find the term independent of x in the expansion of $\left(2x^2 - \frac{3}{x}\right)^9$ 3

d) A window is made by joining a rectangle to an equilateral triangle with dimensions as shown. The perimeter of the window is 18 metres.



NOT TO SCALE

(i) Show that the area (A) of the window is given by 2

$$A = 18x - x^2(6 - \sqrt{3})$$

(ii) Hence find the dimensions of the window that would allow the maximum amount of light to enter the window. Give your answer to the nearest centimetre. 4

End of Question 9

Question 10 (12 marks)

a) Let $f(x) = log_e[(x-3)(5-x)]$. What is the domain of f(x)? 2

b) The position of a particle moving along the x-axis is given by $x = t^2e^{2-t}$, where x is in metres and t is time in seconds.

(i) Find an expression for the velocity in terms of time 1

(ii) Show that the particle is initially at rest 1

(iii) Find the time for which the particle is next at rest 1

(iv) What happens to the particle as time increases indefinitely? 1

c) Melvin the Martian has an infinite number of purple, green, black and yellow socks in a drawer. 1
If Melvin is pulling out socks in the dark, what is the smallest number of socks that Melvin must pull out of the drawer to guarantee getting ten socks of the same colour?

d) A can of soft drink at temperature T degrees is removed from a fridge and placed in a room that has a constant temperature of A degrees Celsius. The rate at which the can soft drink warms can be expressed using the equation

$\frac{dT}{dt} = -k(T-A)$ where t is the time in minutes after the can is placed in the room, and k is a positive constant.

(i) Show that $T = A + Pe^{-kt}$ satisfies the equation, where P is a constant 1

(ii) If the room temperature is 30°C and the soft drink warms from 3°C to 15°C in the first 10 minutes, find the exact value of k 2

(iii) Find the time taken for the temperature of the can to increase by another 12°C (answer to the nearest minute) 2

End of Question 10

Question 11 (12 marks)

a) The polynomial equation $4x^3 - 12x^2 + 5x + 6 = 0$ has roots α, β and γ. 3

It is known that one of the roots is the sum of the other two.

Find the value of α, β and γ

b) Consider the function $f(x) = x^4 - 2x^2$

(i) Find the coordinates of any stationary points and determine their nature 3

(ii) Draw a neat sketch $y = f(x)$, showing the x and y-intercepts and the coordinates of the stationary points 2

(iii) Find the largest domain containing the origin for which $f(x)$ has an inverse function, $f^{-1}(x)$ 1

(iv) State the domain of $f^{-1}(x)$ 1

(v) Find the gradient of $y = f^{-1}(x)$ at $x = -\frac{1}{2}$ 2

End of Question 11

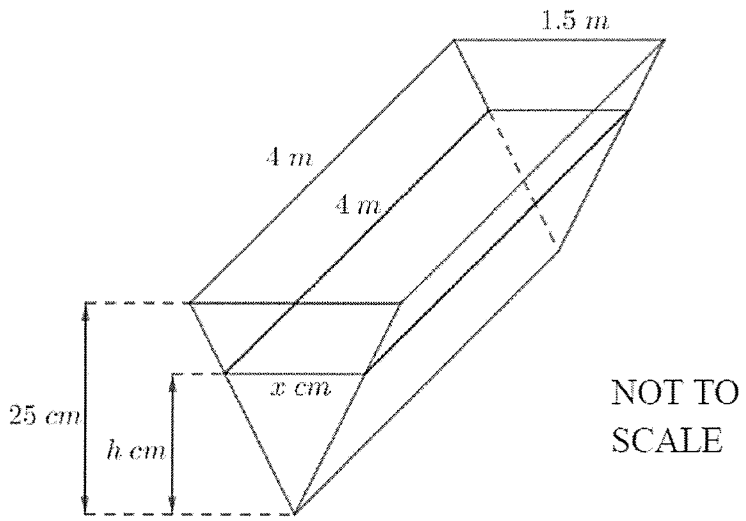
Question 12 (12 marks)

a) Find $\frac{d}{dx}\left(ln\left(x^{\frac{1}{x}}\right)\right)$ for $x > 0$ 2

b) How many arrangements of the word 'MAMMOTH' can be made if only 5 letters are used? 2

c) An open flat topped water trough in the shape of a triangular prism is being emptied through a hole in its base at a constant rate of $18\ 000\ cm^3$ per second. Its top measures 1.5 metres by 4 metres and its triangular end has a vertical height of 25 centimetres.

When the depth is h centimetres, the water surface measures x centimetres by 4 metres.



(i) Use similar triangles to show that when the water depth is h centimetres, the volume $V\ cm^3$ of water in the trough is given by $V = 1200h^2$ 2

(ii) Find the rate at which the depth of water is changing when $h = 20\ cm$ 2

d) Consider the binomial expansion

$$1 + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \cdots + \binom{n}{n}x^n = (1+x)^n$$

Show that, if n is even:

$$4 \times 1 \times \binom{n}{2} + 8 \times 3 \times \binom{n}{4} + 12 \times 5 \times \binom{n}{6} + \cdots + 2n(n-1)\binom{n}{n} = n(n-1)(2)^{n-2}$$ 4

End of Question 12

End of Examination ☺

Mathematics Extension 1 Term 4 2020 solutions

1. $P(2) = 0$

$2^3 - 4(2)^2 - 6(2) + k = 0$

$-20 + k = 0$

$k = 20$ (C)

2. $4\log_e \sqrt{e^x} = \log_e e^{\frac{1}{2}x \times 4}$

$= \log_e e^{2x}$

$= 2x$ (B)

3. $6! \times 2! = 1440$ (D)

4. $\frac{dP}{dt} = 0.2(P-10)$

$P = B + Ae^{kt}$ $B = 10, k = 0.2$

$P = 10 + Ae^{0.2t}$

when $t = 0, P = 30$

$30 = 10 + Ae^0$

$A = 20$

$\therefore P = 10 + 20e^{0.2t}$ (A)

5. $f(x) = e^{x-1}$

$y = e^x - 1$

Inverse: $x = e^y - 1$

$x + 1 = e^y$

$\ln(x+1) = \ln(e^y)$

$\ln(x+1) = y$

$y = \ln(x+1)$

$\frac{dy}{dx} = \frac{1}{x+1}$ (B)

6. $A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$ $\frac{dA}{dt} = 0.1 m^2/min$

$\frac{dr}{dt} = \frac{dr}{dA} \times \frac{dA}{dt}$

$= \frac{1}{2\pi r} \times 0.1$

when $r = 0.3$,

$\frac{dr}{dt} = \frac{0.1}{2\pi(0.3)}$

$= 0.05305\ldots$ (D)

7. (A)

Question 8

a)(i) $y = e^{x}\log_e(2x)$ $u = e^{x}$ $v = \log_e(2x)$

$\frac{du}{dx} = e^{x}$ $\frac{dv}{dx} = \frac{1}{x}$

$y' = e^{x}\log_e(2x) + \frac{e^{x}}{x}$

(ii) $y = 6^{5x}$

$y' = 5.\ln 6.\, 6^{5x}$

(iii) $y = \ln\left(\frac{4x+1}{3x-2}\right)$

$y = \ln(4x+1) - \ln(3x-2)$

$y' = \frac{4}{4x+1} - \frac{3}{3x-2}$

$= \frac{4(3x-2) - 3(4x+1)}{(4x+1)(3x-2)}$

$= \frac{12x - 8 - 12x - 3}{(4x+1)(3x-2)}$

$= \frac{-11}{(4x+1)(3x-2)}$

b)(i) $8! = 40\,320$

(ii) $P(\text{Harry and Ron sit together}) = \frac{2!7!}{8!}$

$= \frac{1}{4}$

$\therefore P(\overline{\text{Harry and Ron sit together}}) = 1 - \frac{1}{4}$

$= \frac{3}{4}$

c) $3^{x-1} = 5$

$(x-1)\ln 3 = \ln 5$

$x = \frac{\ln 5}{\ln 3} + 1$

$x = 2{\cdot}46$ (2 d.p.)

d) $(n+2)! = 72n!$

$(n+2)(n+1)n! = 72n!$

$(n+2)(n+1) = 72$

$n^2 - 3n + 2 = 72$

$n^2 - 3n - 70 = 0$

$(n-7)(n+10) = 0$

$n = 7,\ n = -10$

but $n \geq 0$ $\therefore n = 7$

Question 9

a) $\frac{d}{dx}(\log_7 x^2) = \frac{d}{dx}\left(\frac{\log_e(x^2)}{\log_e 7}\right)$

$= \frac{1}{\log_e 7} \cdot \frac{2}{x}$

$= \frac{2}{x\log_e 7}$

b)(i) ${}^{20}C_5 = 15\,504$

(ii) ${}^{10}C_2 \times {}^{10}C_3 + {}^{10}C_3 \times {}^{10}C_2$

$= 10\,800$

c) $\left(2x^2 - \frac{3}{x}\right)^9$

General term: $T_{r+1} = \binom{n}{r} x^{n-r} a^r$

$T_{r+1} = \binom{9}{r}(2x^2)^{9-r}(-3x^{-1})^r$

$= \binom{9}{r} 2^{9-r}(-3)^r x^{18-2r-r}$

$= \binom{9}{r} 2^{9-r}(-3)^r x^{18-3r}$

Term independent of x:

$18 - 3r = 0$

$r = 6$

$T_7 = \binom{9}{6} 2^{9-6}(-3)^6 x^0$

$= 489\,888$

d)(i) $P = 6x + 2y$ $\quad P = 18$

$6x + 2y = 18$

$2y = 18 - 6x$

$y = 9 - 3x$

$A = 2xy + \frac{1}{2}(2x)(2x)\sin 60°$

$= 2xy + 2x^2 \cdot \frac{\sqrt{3}}{2}$

$= 2xy + \sqrt{3}x^2$

$= 2x(9-3x) + \sqrt{3}x^2$

$= 18x - 6x^2 + \sqrt{3}x^2$

$= 18x - x^2(6 - \sqrt{3})$

(ii) $\frac{dA}{dx} = 18 - 12x + 2\sqrt{3}x$

Maximum amount of light occurs when $\frac{dA}{dx} = 0$

i.e. $18 - x(12 - 2\sqrt{3}) = 0$

$x = \frac{18}{12 - 2\sqrt{3}}$

$= \frac{9}{6 - \sqrt{3}}$

Check nature:

$\frac{d^2A}{dx^2} = -12 + 2\sqrt{3}$

$= -8.5 < 0$

$\therefore$ Maximum when $x = \frac{9}{6-\sqrt{3}}$

Dimensions:

$2x = 2\left(\frac{9}{6-\sqrt{3}}\right)$

$= 4.22\text{m}$

$= 422\text{cm}$

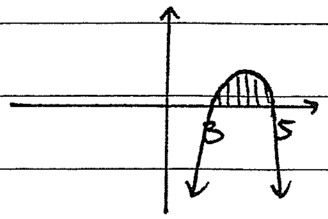
$y = 9 - 3\left(\frac{9}{6-\sqrt{3}}\right)$

$= 2.6739\text{m}$

$= 267\text{ cm}$

Question 10

a) $(x-3)(5-x)>0$



Domain: $3<x<5$

b)(i) $x=t^2e^{2-t}$ $u=t^2$ $v=e^{2-t}$

$\frac{du}{dt}=2t$ $\frac{dv}{dt}=-e^{2-t}$

$v=-t^2e^{2-t}+2te^{2-t}$

(ii) when $t=0$, $v=-0^2 \cdot e^{2-t}+2(0)e^{2-0}$

$v=0$

$\therefore$ initially at rest

(iii) at rest when $v=0$

$-t^2e^{2-t}+2te^{2-t}=0$

$te^{2-t}(-t+2)=0$

$t=0,\ t=2$

$\therefore$ next at rest when $t=2$

(iv) as $t\to\infty$, $x\to 0$

c) With 9 purple, 9 green, 9 black and 9 yellow socks, Melvin still does not have 10 socks of a single colour.

Current total $=9\times 4$

$=36$

$\therefore$ The 37th sock is needed to meet the required condition.

d)(i) $T=A+Pe^{-kt}$

$\frac{dT}{dt}=-k.Pe^{-kt}$

$=-k(T-A)$ as $Pe^{-kt}=T-A$

(ii) $t=0, T=3$ $A=30$

$3=30+Pe^{0}$

$P=-27$

$T=30-27e^{-kt}$

$t=10, T=15$

$15=30-27e^{-10k}$

$15=27e^{-10k}$

$\frac{15}{27}=e^{-10k}$

$\ln\left(\frac{15}{27}\right)=\ln\left(e^{-10k}\right)$

$\ln\left(\frac{15}{27}\right)=-10k$

$k=-\frac{1}{10}\ln\left(\frac{15}{27}\right)$

10.d)(iii) Temperature currently 15°C.

∴ Will increase to $15+12=27°C$

$27=30-27e^{-kt}$

$3=27e^{-kt}$

$\frac{3}{27}=e^{-kt}$

$\ln\left(\frac{1}{9}\right)=\ln\left(e^{-kt}\right)$

$\ln\left(\frac{1}{9}\right)=-kt$

$t=\frac{\ln\left(\frac{1}{9}\right)}{-k}$

$t=\frac{\ln\left(\frac{1}{9}\right)}{-\left(-\frac{1}{10}\ln\left(\frac{15}{27}\right)\right)}$

$t \doteq 37.38$

∴ Time taken to increase by another 12°C is $37.38-10$

$=27$ minutes

Question 11

a) $4x^3-12x^2+5x+6=0$

$\alpha+\beta+\gamma=\frac{-b}{a}$

$\alpha+\beta+\gamma=3$ — ①

$\alpha\beta+\alpha\gamma+\beta\gamma=\frac{5}{4}$ — ②

$\alpha\beta\gamma=-\frac{3}{2}$ — ③

One root is the sum of the other two:

$\alpha=\beta+\gamma$ — ④

sub ④ into ①

$\alpha+\alpha=3$

$2\alpha=3$

$\alpha=\frac{3}{2}$

sub $\alpha=\frac{3}{2}$ into ③

$\frac{3}{2}\beta\gamma=\frac{-3}{2}$

$\beta\gamma=-1$ — ⑤ and $\beta+\gamma=\frac{3}{2}$ — ⑥

From ⑥, $\beta=\frac{3}{2}-\gamma$

sub into ⑤

$\gamma\left(\frac{3}{2}-\gamma\right)=-1$

$\frac{3}{2}\gamma-\gamma^2=-1$

$\gamma^2-\frac{3}{2}\gamma-1=0$

$2\gamma^2-3\gamma-2=0$

$(\gamma-2)(2\gamma+1)=0$

$\gamma=2,\ \gamma=-\frac{1}{2}$

$\beta=\frac{3}{2}-2$ or $\beta=\frac{3}{2}--\frac{1}{2}$

$=-\frac{1}{2}$ $\quad$ $=2$

$\therefore \alpha=\frac{3}{2},\ \beta=-\frac{1}{2},\ \gamma=2$

or $\alpha=\frac{3}{2},\ \beta=2,\ \gamma=-\frac{1}{2}$

11. b) (i) $f(x) = x^4 - 2x^2$

$f'(x) = 4x^3 - 4x$

Stationary points occur when $f'(x)=0$

i.e. $4x^3 - 4x = 0$

$4x(x^2-1) = 0$

$x = 0, \pm 1$

when $x=0, y=0$

when $x=1, y=-1$

when $x=-1, y=-1$

$\therefore$ Stationary points at $(0,0), (1,-1)$ & $(-1,-1)$

Determine nature:

$f''(x) = 12x^2 - 4$

at $x=0$, $f''(0) = -4 \quad <0$

$\therefore$ Maximum turning point at $(0,0)$

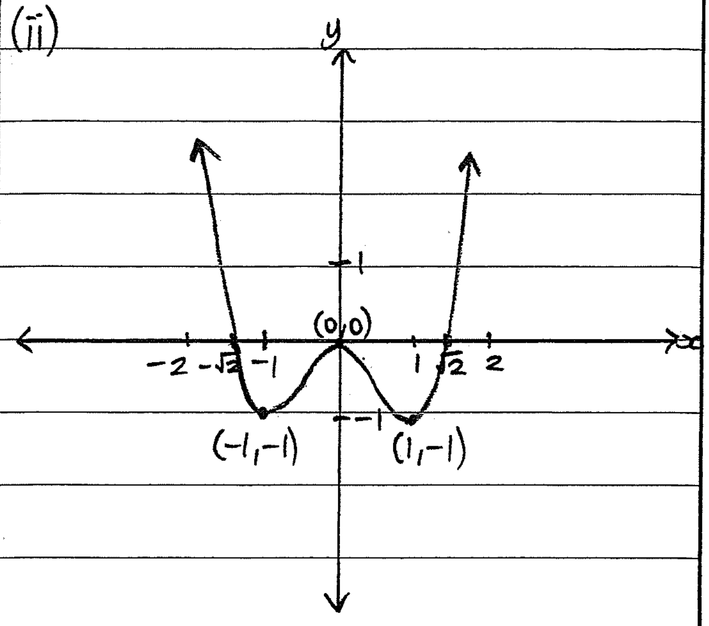
at $x=1$, $f''(1) = 12(1)^2 - 4$
$= 8 \quad > 0$

$\therefore$ Minimum turning point at $(1,-1)$

at $x=-1$, $f''(-1) = 12(-1)^2 - 4$
$= 8 \quad > 0$

$\therefore$ Minimum turning point at $(-1,-1)$

(ii)



x-intercepts: $x^4 - 2x^2 = 0$

$x^2(x^2-2) = 0$

$x = 0, \pm\sqrt{2}$

(iii) Domain: $0 \le x \le 1$

(iv) Restricted range of $y=f(x)$ is $-1 \le y \le 0$

$\therefore$ Domain of $y = f^{-1}(x)$ is $-1 \le x \le 0$

(v) $y = x^4 - 2x^2$

Inverse function: $x = y^4 - 2y^2$

$\frac{dx}{dy} = 4y^3 - 4y$

But $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

$= \frac{1}{4(-\frac{1}{2})^3 - 4(-\frac{1}{2})}$ at $y = -\frac{1}{2}$

$= \frac{1}{-\frac{1}{2}+2}$

$= \frac{2}{3}$

Student Name: ______________________ Teacher Name: ______________________

Question 12

a) $\frac{d}{dx}\left(\ln\left(x^{\frac{1}{x}}\right)\right) = \frac{d}{dx}\left(\frac{1}{x}\ln x\right)$

$u = x^{-1}$ $\quad v = \ln x$

$\frac{du}{dx} = -x^{-2}$ $\quad \frac{dv}{dx} = \frac{1}{x}$

$= \frac{-\ln x}{x^2} + \frac{1}{x^2}$

$= \frac{1}{x^2}(1 - \ln x)$

b) Case 1 (1 M) M _ _ _ _ $= 5!$ ways

Case 2 (2 M's) M M _ _ _ $= \frac{5!}{2!} \times {}^4C_3$

Case 3 (3 M's) M M M _ _ $= \frac{5!}{3!} \times {}^4C_2$

Total arrangements $= 5! + \frac{5!}{2!} \times {}^4C_3 + \frac{5!}{3!} \times {}^4C_2$

$= 480$

c) (i) By similar triangles, $\frac{x}{150} = \frac{h}{25}$

$x = 6h$

$V = \frac{1}{2} \times x \times h \times 400$

$= \frac{1}{2} \times 6h \times h \times 400$

$= 1200h^2$

(ii) $\frac{dh}{dt} = ?$

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ $\quad \frac{dV}{dt} = -18\,000 \text{ cm}^3/\text{s}$

$V = 1200h^2$

$\frac{dV}{dh} = 2400h$

12.(ii) continued

$\frac{dh}{dV} = \frac{1}{2400h}$

when $h = 20\text{cm}$,

$\frac{dh}{dt} = \frac{1}{2400 \times 20} \times -18000$

$= \frac{-3}{8}$

$= -0.375 \text{ cm/s}$

$\therefore$ the water is falling at a rate of 0.375 cm/s

d) $1 + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \ldots + \binom{n}{n}x^n = (1+x)^n$

Differentiate both sides w.r.t x

$\binom{n}{1} + 2\binom{n}{2}x + 3\binom{n}{3}x^2 + 4\binom{n}{4}x^3 + \ldots + n\binom{n}{n}x^{n-1} = n(1+x)^{n-1}$

Differentiate both sides w.r.t x

$2\binom{n}{2} + 2\times3\binom{n}{3}x + 3\times4\binom{n}{4}x^2 + \ldots + n(n-1)\binom{n}{n}x^{n-2} = n(n-1)(1+x)^{n-2}$

Let $x = 1$

$2\binom{n}{2} + 2\times3\binom{n}{3} + 3\times4\binom{n}{4} + \ldots + n(n-1)\binom{n}{n} = n(n-1)(2)^{n-2}$ —①

Let $x = -1$

$2\binom{n}{2} - 2\times3\binom{n}{3} + 3\times4\binom{n}{4} - \ldots + n(n-1)\binom{n}{n}(-1)^{n-2} = 0$ —②

$(-1)^{n-2} = 1$ since n is even

Adding ① and ②

$\therefore 4\times1\binom{n}{2} + 8\times3\binom{n}{4} + 12\times5\binom{n}{6} + \ldots + 2n(n-1)\binom{n}{n} = n(n-1)(2)^{n-2}$

as required.

Mathematics Extension 1 Marker Comments

Question 8

a) i. Well done apart from some carelessness in using the product rule

a) ii. Poorly done even though this could be done using the formula sheet

a) iii. Some students reduced this to the quotient rule and ignored the necessity of using the logarithm derivative

b) i. Some students answered 9! Instead of 8! . This was the circular problem!

b) ii. Many students gave the number of arrangements rather than the required probability

c) Some students have difficulty using the log laws

d) Well done apart from some students having difficulty finding the required quadratic equation. Many did not dismiss -10 as a possible solution.

Question 9

a) Students should be aware of less standard derivatives, which are still on the formula sheet.

b) Where i) was well done, there were several common errors in ii), as it was a tricky question. The most common mistake was ${}^{10}C_2 \times {}^{10}C_2 \times 16$. This significantly overcounted, such that it was larger than the answer in i).

c) The two most successful methods included the use of the general term and the (inefficient) expansion of the entire binomial. In both cases, students should ensure that they are answering the original question. Other common errors included leaving a negative, and not matching the powers to the instance of ${}^9C_k$.

d) i) This was generally well done, whether students used the sine rule for the area of the triangle, or found the perpendicular height separately. Students should work on clearly (and efficiently) showing their working.

ii) This is a fairly standard optimisation, and students should ensure that they are familiar with the requirements. A test to verify that you have found a maximum is always required. Students should also ensure that they answer the question: finding the **dimensions** of $2x$ and y. Moreover, many students were confused by answering the question to the nearest centimetre, as the dimensions are in metres.

Question 10

a) Some students cannot solve $(x-3)(5-x) > 0$. The most common incorrect answer was $x > 5 \; or \; x < 3$

b) (i) most students cannot do product rule when differentiating $x = t^2e^{2-t}$ with respect to t

c) Some students cannot apply pigeonhole principle

d) (ii) some students could not find the value of k as they cannot solve a logarithmic equation.

(iii) Some students could not get the correct answer as they cannot solve a logarithmic equation

Question 11

a) Most were able to show that one root is the sum of the other two but some incorrectly found the sum of the roots one at a time, sum of roots two at a time and product of the roots – these are on the reference sheet! Some students found one root and used long division to find the other two roots – great alternative method to solving simultaneously.

b) (i) This is a typical Advanced Mathematics question and many lost marks. Some students forgot that solving a quadratic equation gives two solutions, $x = \pm 1$. You must check the nature of stationary points either using the first derivative table or substituting into the second derivative. Coordinates of stationary points means you need to find the y-values too!

(ii) Read the question - you were asked to show the intercepts.

(iii) To have an inverse function, there must be a 1-1 relationship between the x and y values i.e. the vertical and horizontal line must both be passed. Also the origin must be contained.

(iv) Remember that the domain and range from the original function to the inverse swap. So the range from the domain restriction in part iii should be considered to find the domain of the inverse.

(v) Poorly done overall. Those who realised to differentiate the inverse function with respect to y and find the reciprocal of $\frac{dx}{dy}$ before substituting in $y = -\frac{1}{2}$ had most success.

Question 12

a) Many failed to use the log laws to simplify in order to be able to differentiate – we cannot subtract one from a variable power, only a numerical power

b) Had to consider cases – but you don't have to choose from the 3 M's, they ARE THE SAME LETTER

c) i. Many forgot to make the units the same – change all to cm or to m BUT NOT A MIX OF BOTH – you do not need to prove similarity in these cases

c) ii. The rate was decreasing – so it should have negative (not penalised) – some forgot that $\frac{a}{h}$ does not become ah

d) Many non attempts – successful students subbed both $x = 1$ and $x = -1$ and added the 3 expressions together, OR recognised that for even values of n, the sum of the odd coefficients is equal to the sum of the even coefficients (PASCAL's TRIANGLE) – just writing "there is symmetry" was not enough you had to justify what kind of symmetry and how to use it.